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Internal heat–mass transfer and stresses in thin-walled structures of ablating materials

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Abstract—On the basis of the theory of internal heat–mass transfer and deforming for ablating materials developed before the present paper, a theory of heat–mass transfer processes and stress state in thin-walled shell structures made of ablating composite materials is suggested. This theory is based on the system of special hypotheses with respect to distributions of main process characteristics vs the shell thickness. Accurate analytical solution is obtained for cylindrical ablating shells. Comparison of the ablating shells' theory suggested with results of computations shows a high accuracy of the theory up to 15%, that is a fine result for a shell theory. A theory of delaminations' appearance in ablating shells and their stability loss under the action of internal pore gas pressure is developed. Calculations of stability estimation for tank-containers made of ablating glass–plastics for the carriage of harsh media in fire.

1. INTRODUCTION

At present thin-walled shell structures made of composite materials have a wide application in different fields of technique: from aerospace structures to tank-containers for the carriage of harsh media. Up to now, design of such hardware has been based in the main on strength calculations [1]. However, nowadays investigation problems on behaviour of thin-walled structures made of polymer composite materials under conditions of high temperatures' action become more and more realistic, for example, this problem appears in the investigation of the resistance of hardware of the tank-container type in fire or in calculation of aerospace structures' work-capacity in aerodynamic heating.

Up to now, the behaviour of thin-walled structures made of polymer composites under conditions of high temperatures has not been investigated. It is evident that their behaviour essentially differs from the one of metallic thin-walled structures. Due to substantially lower heat-conductivity, polymer composite materials have a longer resource for location in a high temperature zone. However, there is an internal ablation (pyrolise) in composites at high temperatures that is accompanied by intensive gas generation. Gases in pores have no time to be filtrated to the external surface and create excess pore pressure that can lead to delamination of the glass–plastic shell structure. After appearance of the delamination, a closed shell structure of the cylindrical type is still capable to keep stability to the action of high temperatures for certain times and only when internal gas pressure in a cavity formed by delamination reaches its critical value, then thermomechanical destruction of the whole structure occurs due to a loss of stability of its internal layers. Possibility of quantitative description of these phenomena is extremely important for estimation of a stability resource of thin-walled structures under conditions of the high temperatures' action. Development of such methods of calculation is the objective of this paper.

The ablating composite material model describing coupled processes of internal heat–mass transfer and deforming was developed in refs. [2–5]. In the present paper a theory of heat–mass transfer and stress state is developed on the basis of this model for thin-walled shell structures of ablating glass–plastics taking account of their delamination at the action of high temperatures.

2. GENERAL EQUATIONS

In accordance with ref. [2], a composite material at high temperatures is considered to be a four-phase medium: the first phase is a thermostable reinforcing filler in the form of glass or other fibres; the second phase is a polymer matrix; the third phase is a solid residue of high-temperature pyrolise of polymer; the fourth phase is gaseous products of pyrolise in pores of the material. The first three phases form a monolithic framework of material. Consider the case when a reinforcing filler is oriented in such manner that the composite as a whole can be considered as an orthotropic laminated material with orthotropy axes being coincident with the axes of the chosen curvilinear orthogonal coordinate system Oq_α, $\alpha = 1, 2, 3$ and the Oq_3 axis is orthogonal to the plane of the composite layers and to the shell surface.

Write a general system of equations for a composite

NOMENCLATURE

A_α coefficients of the first squared form of a middle surface of a shell
$b_{i\kappa}$ material constants describing a change of strength and elastic features of composites with temperature
c_g, c_i specific heat-capacities of phases $[\mathrm{m^2(s^2\,K)^{-1}}]$
$e_{\alpha\beta}$ strains of a shell
E_i elasticity modules of phases $[\mathrm{kg\,m^{-1}\,s^{-2}}]$
$\Delta e_g^0, \Delta e_g^*$ heat of volumetric and surface ablation $[\mathrm{m^2\,s^{-1}}]$
$f(\bar{x}, t)$ shape of phase separation surface
h shell thickness [m]
h_d thickness of a nondelaminated part of a shell
$H_\alpha\, \alpha = 1,2,3$; Lamet's parameters of curvilinear coordinate system q_α
J intensity of mass transfer from polymer phase to gas $[\mathrm{kg(m^3\,s)^{-1}}]$
$M_{\alpha\beta}$ moments in a shell $[\mathrm{kg\,m\,s^{-2}}]$
p_e gas pressure on an external composite surface $[\mathrm{kg(m\,s^2)^{-1}}]$
p pore gas pressure $[\mathrm{kg(m\,s^2)^{-1}}]$
p_d gas pressure in a delamination cavity of a shell
Q_α crossing forces $[\mathrm{kg\,s^{-2}}]$
q_α curvilinear orthogonal coordinates
R radius of a middle surface of a cylindrical shell [m]
R_a universal gas constant $[\mathrm{m^2(s^2\,K)^{-1}}]$
t time [s]
t_* the time of appearance of the first shell delamination
t_{**} the time of stability loss of a shell
$T_{\alpha\beta}$ forces in a shell
U_α displacement of a middle surface of a shell [m]
W deflection of a shell [m]
z_1 damage parameter of a shell.

Greek symbols
$\alpha_i, \alpha_{\kappa i}$ coefficients of heat phase expansion $[\mathrm{K^{-1}}]$
α^T coefficient of heat transfer $[\mathrm{kg(s^3\,K)^{-1}}]$
β_{sh} shrinkage coefficient
Γ gasification coefficient
$\boldsymbol{\varepsilon}, \varepsilon_{ij}$ strain tensor and its components
θ temperature [K]
λ_{ij} components of a heat-conduction tensor $[\mathrm{kg\,m(s^3\,K)^{-1}}]$
ν_{ij} Poisson coefficients
ρ_g, ρ_i phase density $[\mathrm{kg\,m^{-3}}]$
σ_{ij} stress tensor's components $[\mathrm{kg(m\,s^2)^{-1}}]$
$\kappa_{\alpha\beta}$ curvatures of a shell.

Subscripts
e (external) parameters of the surroundings
g (gas) parameters of a gas phase.

with internal ablation in this coordinate system. Then equilibrium equations have the following form:

$$\frac{\partial}{\partial q_\alpha}(H_\beta H_\gamma \sigma_{\alpha\alpha}\phi_s) + \frac{\partial}{\partial q_\beta}(H_\alpha H_\gamma \sigma_{\alpha\beta}\phi_s) + \frac{\partial}{\partial q_\gamma}(H_\alpha H_\beta \sigma_{\alpha\gamma}\phi_s) - \phi_s\sigma_{\beta\beta}H_\gamma\frac{\partial H_\beta}{\partial q_\alpha} - \phi_s\sigma_{\gamma\gamma}H_\beta\frac{\partial H_\gamma}{\partial q_\alpha} + \phi_s\sigma_{\alpha\beta}H_\gamma\frac{\partial H_\alpha}{\partial q_\beta} + \phi_s\sigma_{\alpha\gamma}H_\beta\frac{\partial H_\alpha}{\partial q_\gamma} - H_\beta H_\gamma\frac{\partial \phi_g p}{\partial q_\alpha} = 0;$$
$$\alpha, \beta, \gamma = 1,2,3; \quad \alpha \neq \beta \neq \gamma; \tag{1}$$

equations of changing mass of the second and third phases are

$$\rho_3^0\frac{\partial \phi_3}{\partial t} = (1-\Gamma)J; \tag{2}$$

$$\rho_2^0\frac{\partial \phi_2}{\partial t} = -J; \tag{3}$$

equation of filteration of pyrolise gaseous products in pores is

$$\frac{\partial \rho_g}{\partial t} = \frac{1}{H_1H_2H_3}\left(\frac{\partial}{\partial q_1}\left(\frac{H_2H_3}{H_1}K_{11}R_a\frac{\partial \rho_g\theta}{\partial q_1}\right) + \frac{\partial}{\partial q_2}\left(\frac{H_1H_3}{H_2}K_{22}R_a\frac{\partial \rho_g\theta}{\partial q_2}\right) + \frac{\partial}{\partial q_3}\left(\frac{H_1H_2}{H_3}K_{33}R_a\frac{\partial \rho_g\theta}{\partial q_3}\right)\right) + \Gamma J \tag{4}$$

and equation of heat transfer is

$$\rho c\frac{\partial \theta}{\partial t} = \frac{1}{H_1H_2H_3}\left(\frac{\partial}{\partial q_1}\left(\frac{H_2H_3}{H_1}\lambda_{11}\frac{\partial \theta}{\partial q_1}\right) + \frac{\partial}{\partial q_2}\left(\frac{H_1H_3}{H_2}\lambda_{22}\frac{\partial \theta}{\partial q_2}\right) + \frac{\partial}{\partial q_3}\left(\frac{H_1H_2}{H_3}\lambda_{33}\frac{\partial \theta}{\partial q_3}\right)\right) + \frac{c_4K_{11}R}{H_1^2}\frac{\partial \rho_g\theta}{\partial q_1}\frac{\partial \theta}{\partial q_1}$$

$$+\frac{c_4K_{22}R}{H_2^2}\frac{\partial\rho_g\theta}{\partial q_2}\frac{\partial\theta}{\partial q_2}+\frac{c_4K_{33}R}{H_3^2}\frac{\partial\rho_g\theta}{\partial q_3}\frac{\partial\theta}{\partial q_3}-\Delta e^0 J, \tag{5}$$

here H_1, H_2 and H_3 are Lamet's parameters [6]. The Cauchy's relations connected composite strains $\varepsilon_{\alpha\beta}$ with displacements u_α in the curvilinear coordinate system have the form:

$$\varepsilon_{\alpha\alpha}=\frac{1}{H_\alpha}\frac{\partial u_\alpha}{\partial q_\alpha}+\frac{1}{H_\alpha H_\beta}\frac{\partial H_\alpha}{\partial q_\beta}u_\beta+\frac{1}{H_\alpha H_\gamma}\frac{\partial H_\alpha}{\partial q_\gamma}u_\gamma; \tag{6}$$

$$2\varepsilon_{\alpha\beta}=\frac{H_\alpha}{H_\beta}\frac{\partial}{\partial q_\beta}\left(\frac{u_\alpha}{H_\alpha}\right)+\frac{H_\beta}{H_\alpha}\frac{\partial}{\partial q_\alpha}\left(\frac{u_\beta}{H_\beta}\right);$$

$$\alpha,\beta,\gamma=1,2,3;\quad \alpha\neq\beta\neq\gamma. \tag{7}$$

Constitutive relations for an orthotropic laminated ablating material have the form:

$$\sigma_{\alpha\alpha}=-p+\tilde{a}_1^0\sum_{\beta=1}^{3}C_{\alpha\beta}(\varepsilon_{\beta\beta}-\mathring{\varepsilon}_{\beta\beta});\quad \alpha=1,2; \tag{8}$$

$$\sigma_{33}=-p+\tilde{a}_1^0\sum_{\beta=1}^{3}C_{3\beta}(\varepsilon_{\beta\beta}-\mathring{\varepsilon}_{\beta\beta})+\tilde{a}_2^0C_{33}(\varepsilon_{33}-\mathring{\varepsilon}_{33}); \tag{9}$$

$$\sigma_{12}=\tilde{a}_1^0C_{66}\varepsilon_{12}; \tag{10}$$

$$\sigma_{23}=\tilde{a}_2^0C_{55}\varepsilon_{23}; \tag{11}$$

$$\sigma_{13}=\tilde{a}_2^0C_{44}\varepsilon_{13} \tag{12}$$

where p is pore pressure of gas:

$$p=\rho_gR_a\theta, \tag{13}$$

and $C_{\alpha\beta}$ are elasticity modules of orthotropic material. $\tilde{a}_1^0$ and $\tilde{a}_2^0$ are functions describing a change of elastic features of the composite at raised temperatures up to pyrolise temperatures:

$$\tilde{a}_2^0=\left(1-\frac{\phi_2^0}{b_2}+\frac{\phi_2+\phi_3}{b_2\phi_2+b_3\phi_3}\right)^{-1}, \tag{14}$$

$$\tilde{a}_1^0=1+a_2(\phi_2-\phi_2^0)+a_2\phi_3,\quad b_2=\phi_2^0+\frac{(1-\phi_2^0)^2}{1/a_2-\phi_2^0};$$

$$b_3=b_2a_3/a_2, \tag{15}$$

where a_2 and a_3 are the model constants determined in experiments [3]; ϕ_2^0 is the initial concentration of a binder in the composite.

Heat deformations $\mathring{\varepsilon}_{\beta\beta}$ of an ablating composite consist of three terms: heat expansion of the material, deformation of phase transformation (pyrolise) shrinkage in pyrolise [2].

$$\mathring{\varepsilon}_{\beta\beta}=\frac{1}{a_1^0}\Bigg((\alpha_1(1-a_2\phi_2^0)+\alpha_2\phi_2\mathrm{a}_2)(\theta-\theta_0)+\alpha_3\mathrm{a}_3\int_0^t\theta\dot{\phi}_3\mathrm{d}\tau-\beta_{sh}\mathrm{a}_3\phi_3\Bigg);$$

$$\mathring{\varepsilon}_{33}=(\alpha_1(1-\phi_2^0)+\alpha_2\phi_2)(\theta-\theta_0)+\alpha_3\int_0^t\theta\dot{\phi}_3\,\mathrm{d}\tau-\beta_{sh}\phi_3,$$

$$\beta=1,2; \tag{16}$$

here α_i are coefficients of heat expansion of the ith phase, β_{sh} is the shrinkage of the binder in pyrolise.

3. EQUATIONS OF THIN-WALLED ABLATING SHELLS

Let us consider a thin-walled shell of ablating material with the thickness h, where the line q_3 coincides with the normal to the surface, $-(h/2)\leqslant q_3\leqslant(h/2)$ and q_1 and q_2 coincide with the lines of main curvatures of the shell surface. There are $H_3=1$, $H_\alpha=A_\alpha(1+k_\alpha q_3)$ [6] for this case, where $A_\alpha(q)$ are coefficients of the first square form of the surface reduced ($q_3=0$), k_α are its main curvatures.

Boundary conditions for the equation system (1)–(7) at surfaces $q_3=\pm h/2$ have the form:

$$\phi_s\sigma_{33}-\phi_gp=-p_\pm,\quad \sigma_{\alpha3}=0;$$

$$\mp\lambda_{33}\frac{\partial\theta}{\partial q_3}=q_\pm-(\theta-\theta_\pm)R_ac_gK_{33}\frac{\partial\rho_g\theta}{\partial q_3};$$

$$\alpha=1,2;\quad p=p_\pm, \tag{17}$$

where p_+ is the external pressure on the shell; $q_\pm$ is the heat flux to the external surfaces.

Let us consider very thin shells for which there is $k_\alpha q_3\ll1$ then the terms $k_\alpha q_3$ can be neglected in comparison with 1 for $\alpha=1,2$; in particular $H_\alpha\approx A_\alpha$ for $\alpha=1,2$.

Forces T_{11}, T_{12}, T_{22}, moments M_1, M_2 and M_{12}, crossing forces Q_1, Q_2, averaged pressure P_g are introduced as follows:

$$T_{\alpha\beta}=\frac{1}{\bar{\phi}_s}\int_{-h/2}^{h/2}\phi_s\sigma_{\alpha\beta}\,\mathrm{d}q_3;\quad Q_\alpha=\frac{1}{\bar{\phi}_s}\int_{-h/2}^{h/2}\phi_s\sigma_{\alpha3}H_2\,\mathrm{d}q_3;$$

$$M_{\alpha\beta}=\frac{1}{\bar{\phi}_s}\int_{-h/2}^{h/2}\phi_s\sigma_{\alpha\beta}q_3\,\mathrm{d}q_3;\quad M_g=\frac{1}{\bar{\phi}_g}\int_{-h/2}^{h/2}\phi_gp_gq_3\,\mathrm{d}q_3;$$

$$P_g=\frac{1}{\bar{\phi}_s}\int_{-h/2}^{h/2}\phi_gp_g\,\mathrm{d}q_3;\quad \bar{\phi}_s\equiv\frac{1}{h}\int_{-h/2}^{h/2}\phi_s\,\mathrm{d}q_3;$$

$$\phi_g=1-\phi_s,\quad \alpha,\beta=1,2. \tag{18}$$

Averaging the equilibrium equation (1) over the thickness, we obtain:

$$\frac{\partial\bar{\phi}_sA_2T_{11}}{\partial q_1}+\frac{\partial\bar{\phi}_sA_1T_{21}}{\partial q_2}-\frac{\partial A_2}{\partial q_1}T_{22}\bar{\phi}_s+\frac{\partial A_1}{\partial q_2}\bar{\phi}_sT_{12}+\bar{\phi}_sA_1A_2k_1Q_1-A_2\frac{\partial\bar{\phi}_gP_g}{\partial q_1}=0;$$

$$\frac{\partial\bar{\phi}_s A_1 T_{22}}{\partial q_2}+\frac{\partial\bar{\phi}_s A_2 T_{12}}{\partial q_1}-\frac{\partial A_1}{\partial q_2}T_{11}\bar{\phi}_s+\frac{\partial A_2}{\partial q_1}\bar{\phi}_s T_{21}+\bar{\phi}_s A_1 A_2 k_2 Q_2-A_1\frac{\partial\bar{\phi}_g P_g}{\partial q_2}=0;$$
$$-A_1A_2\bar{\phi}_s(k_1T_{11}+k_2T_{22})+\frac{\partial A_2 Q_1\bar{\phi}_s}{\partial q_1}+\frac{\partial A_1 Q_2\bar{\phi}_s}{\partial q_2}-p_3A_1A_2-(k_1+k_2)A_2A_1\bar{\phi}_g P_g=0. \quad (19)$$

Multiplying the equation (1) for $\alpha = 1, 2$ by q_3 and integrating over the thickness, we find two equations for the moments:

$$\frac{\bar{\phi}_s A_2 M_{11}}{\partial q_1}+\frac{\partial\bar{\phi}_s A_1 M_{21}}{\partial q_2}+\frac{\partial A_1}{\partial q_2}M_{12}\bar{\phi}_s-\frac{\partial A_2}{\partial q_1}\bar{\phi}_s M_{22}-A_1A_2\bar{\phi}_s Q_1-A_2\frac{\partial\bar{\phi}_g M_g}{\partial q_1}=0; \quad (20)$$

$$\frac{\partial\bar{\phi}_s A_1 M_{22}}{\partial q_1}+\frac{\partial\bar{\phi}_s A_2 M_{12}}{\partial q_2}+\frac{\partial A_2}{\partial q_1}M_{21}\bar{\phi}_s-\frac{\partial A_1}{\partial q_2}\bar{\phi}_s M_{11}-A_1A_2\bar{\phi}_s Q_2-A_1\frac{\partial\bar{\phi}_g M_g}{\partial q_2}=0.$$

At present there exists a great number of theories for calculation of a stress–strain state of thin-walled shells [6–9]. For laminated composite materials the Timoshenko's shell theory is the simplest and sufficiently acceptable. However, this theory and also its analogs cannot be applied to ablating shells as they do not consider the most dangerous delaminating stresses σ_{33} arising due to internal gas generation of an ablating laminate. In order to take account of this effect it is necessary to formulate a new system of hypotheses for a shell theory:

(a) distribution of displacements u_α, u_3 through the shell thickness is chosen in the form

$$u_\alpha=\frac{U_\alpha+q_3\gamma_\alpha}{\tilde{a}_1^0\phi_s};\quad u_3=W; \quad (21)$$

(b) distributions of shear $\sigma_{\alpha 3}$ and normal σ_{33} stresses are chosen in the form

$$\phi_s\sigma_{\alpha 3}=C_{\alpha+3,\alpha+3}\left(-q_3^2+\frac{h^2}{4}\right)f_\alpha, \quad (22)$$

$$\phi_s\sigma_{33}=\phi_g p-p_-+(p_-p_+)\left(\frac{q_3}{h}+\frac{1}{2}\right)+\left(-q_3^2+\frac{h^2}{4}\right)f_3,\quad \alpha=1,2, \quad (23)$$

where U_α, γ_α, W and f_α, f_3 are functions of coordinates q_β and t, $\alpha, \beta = 1, 2$;

(c) the following integral relations are considered instead of state equations (9), (11), (12)

$$\int_{-h/2}^{h/2}\phi_s\sigma_{33}\,dq_3=-\int_{-h/2}^{h/2}\phi_s p\,dq_3+\sum_{\beta=1}^{2}C_{3\beta}\int_{-h/2}^{h/2}\phi_s\tilde{a}_1^0(\varepsilon_{\beta\beta}-\mathring{\varepsilon}_{\beta\beta})\,dq_3+C_{33}\int_{-h/2}^{h/2}\phi_s\tilde{a}_2^0(\varepsilon_{33}-\mathring{\varepsilon}_{33})\,dq_3, \quad (24)$$

$$\int_{-h/2}^{h/2}\phi_s\sigma_{\alpha 3}\,dq_3=C_{\alpha+3,\alpha+3}\int_{-h/2}^{h/2}\phi_s\tilde{a}_2^0\varepsilon_{\alpha 3}\,dq_3,\quad \alpha=1,2; \quad (25)$$

(d) external heating of the shell by heat fluxes $q_\pm$ is considered to be low-changing along the shell surface and thus temperature θ, pore pressure p, gas density ρ_g and concentrations ϕ_g, ϕ_2, ϕ_3 distributions can be assumed to be functions only of coordinate q_3 and time t.

Distributions (21) and (22) satisfy boundary conditions (19) at the shell surfaces $q_3 = \pm h/2$ automatically.

Substituting expressions (20) into the kinematic relations (6) and (7), we obtain the expressions for strains:

$$\varepsilon_{\alpha\beta}=\frac{e_{\alpha\beta}+q_3\kappa_{\alpha\beta}}{\tilde{a}_1^0\phi_s},\quad \varepsilon_{33}=0; \quad (26)$$

$$2\varepsilon_{\alpha 3}=\frac{1}{A_\alpha}\frac{\partial W}{\partial q_\alpha}+\frac{\chi_\alpha\gamma_\alpha-k_\alpha U_\alpha}{\tilde{a}_1^0\phi_s},\quad \alpha=1,2,$$

where the following designations are introduced

$$e_{\alpha\alpha}=\frac{1}{A_\alpha}\frac{\partial U_\alpha}{\partial q_\alpha}+\frac{1}{A_1A_2}\frac{\partial A_\alpha}{\partial q_\beta}U_\beta+k_\alpha W,\quad \alpha=1,2,$$

$$e_{12}=\frac{1}{2A_2}\left(\frac{\partial U_1}{\partial q_2}\right)+\frac{1}{2A_1}\left(\frac{\partial U_2}{\partial q_1}\right)-\frac{1}{2A_1A_2}\left(\frac{\partial A_1}{\partial q_2}U_1+\frac{\partial A_2}{\partial q_1}U_2\right);$$

$$\kappa_{\alpha\alpha}=\frac{1}{A_\alpha}\frac{\partial\gamma_\alpha}{\partial q_\alpha}+\frac{1}{A_1A_2}\frac{\partial A_\alpha}{\partial q_\beta}\gamma\beta;$$

$$\chi_\alpha=1-q_3k_\alpha-\frac{1}{\tilde{a}_1^0\phi_s}\frac{\partial}{\partial q_3}(\tilde{a}_1^0\phi_s), \quad (27)$$

being kinematic relations for thin-walled shells.

Substituting distributions (22) and (23) for stresses and (26) for strains into relations (24) and (25) we derive equations allowing us to express functions f_α, f_3 in terms of γ_α, U_α and W:

$$f_\alpha=\frac{6}{h^3}\left(\bar{\chi}_\alpha\gamma_\alpha+\frac{h}{A_\alpha}\frac{\partial W}{\partial q_\alpha}-\bar{\chi}_0k_\alpha U_\alpha\right);$$

$$f_3 = \frac{6}{h^3}\left(-\phi_s P_s - \phi_g P_g + (p_+ + p_-)\frac{h}{2} - \sum_{\beta=1}^{3} C_{3\beta}\overset{\circ}{\varepsilon}{}^{(0)}_{\beta\beta} + h\sum_{\beta=1}^{2} C_{3\beta}e_{\beta\beta}\right), \quad (28)$$

where the following designations are introduced

$$\bar{\chi}_\alpha = \int_{-h/2}^{h/2} \tilde{a}_2^0 \chi_\alpha \, dq_3 ; \quad \bar{\chi}_0 = \int_{-h/2}^{h/2} \frac{\tilde{a}_2^0}{\tilde{a}_1^0} dq_3 ;$$

$$\overset{\circ}{\varepsilon}{}^{(i)}_{\beta\beta} = \frac{1}{h}\int_{-h/2}^{h/2} \tilde{a}_1^0 \phi_s \overset{\circ}{\varepsilon}_{\beta\beta} q_3^i \, dq_3 ; \quad P_s = \frac{1}{\bar{\phi}_s}\int_{-h/2}^{h/2} \phi_s p \, dq_3 ;$$

$$M_s = \frac{1}{\bar{\phi}_s}\int_{-h/2}^{h/2} p\phi_s q_3 \, dq_3 ; \quad P_g = \frac{1}{\bar{\phi}_g}\int_{-h/2}^{h/2} p\phi_g \, dq_3 . \quad (29)$$

If functions γ_α, U_α and W are known, then stresses $\sigma_{\alpha 3}$ in a shell can be determined by formulae (22) and (23) and $\sigma_{\alpha\beta}$ by formulae:

$$\sigma_{\alpha\alpha} = -p + \frac{1}{\phi_s}\sum_{\beta=1}^{3} C_{\alpha\beta}(e_{\beta\beta} + q_3\kappa_{\beta\beta} - \overset{\circ}{\varepsilon}_{\beta\beta}) ;$$

$$\sigma_{12} = \frac{C_{66}}{\phi_s}(e_{12} + q_3\kappa_{12}). \quad (30)$$

On substituting expressions (21) and (30) into (18), the constitutive relations for a shell can be derived:

$$\bar{\phi}_s T_{\alpha\alpha} = \bar{\phi}_s P_s + \sum_{\beta=1}^{2} C_{\alpha\beta}(he_{\beta\beta} - \overset{\circ}{\varepsilon}{}^{(0)}_{\beta\beta}) - C_{\alpha 3}\overset{\circ}{\varepsilon}{}^{(0)}_{33} ;$$

$$\bar{\phi}_s T_{12} = C_{66} h e_{12} ;$$

$$\bar{\phi}_s M_{\alpha\alpha} = -\bar{\phi}_s M_s + \sum_{\beta=1}^{2} C_{\alpha\beta}\left(\frac{h^3}{12}\kappa_{\beta\beta} - \overset{\circ}{\varepsilon}{}^{(1)}_{\beta\beta}\right) - C_{\alpha 3}\overset{\circ}{\varepsilon}{}^{(1)}_{33} ;$$

$$\bar{\phi}_s M_{12} = \frac{h^3}{12} C_{66}\kappa_{12} ;$$

$$Q_\alpha = \frac{h^3}{6} C_{\alpha+3,\alpha+3}\left(\bar{\chi}_\alpha \gamma_\alpha - \chi_0 k_\alpha U_\alpha + \frac{h}{A_\alpha}\frac{\partial W}{\partial 2_\alpha}\right);$$

$$\alpha = 1, 2. \quad (31)$$

Taking account of the assumption (d), equations of internal heat and mass transfer (2)–(5) for a shell are rewritten in the form:

$$\rho_2^0 \frac{\partial \phi_2}{\partial t} = -J ;$$

$$\frac{\partial \rho_g}{\partial t} = \frac{1}{A_1 A_2}\frac{\partial}{\partial q_3}\left(A_1 A_2 K_{33} R_a \frac{\partial \rho_g \theta}{\partial q_3}\right) + \Gamma J ;$$

$$\rho c \frac{\partial \theta}{\partial t} = \frac{1}{A_1 A_2}\frac{\partial}{\partial q_3}\left(A_1 A_2 \lambda_{33}\frac{\partial \theta}{\partial q_3}\right) + c_g R_a K_{33}\frac{\partial \rho_g \theta}{\partial q_3}\frac{\partial \theta}{\partial q_3} - \Delta e^0 J. \quad (32)$$

Thus, equilibrium equations (19) and (20) are the ones into which constitutive relations (31) and kinematic relations (27) should be substituted. Also equations of heat and mass transfer (32) are the closed system of eight equations used to determine five functions U_α, γ_α, W depending on q_α, t, $\alpha = 1, 2$ and three functions ρ_g, ϕ_2, θ depending on q_α, q_3, t. It should be noted that due to assuming the hypothesis (d), functions ρ_g, ϕ_2 and θ depend on q_α only parametrically: i.e. by the dependence of external heat flux $q_\pm$ and pressure $p_\pm$ at the shell surface upon coordinates q_α in boundary conditions (23).

Contour L bounding the shell according to the assumption (d) should be hermetic and heat-insulated, i.e. at this contour all derivatives should be equal to zero: $\partial\theta/\partial q_\alpha = 0$, $\partial\rho_g\theta/\partial q_\alpha = 0$. Therefore there are only 'mechanical' boundary conditions at the contour L for system (19) and (20). For example, at contour $q_\alpha = \text{const}$ they are given by five values, i.e. by one value from each pair:

$$(\bar{\phi}_s T_{11} - \bar{\phi}_s P_g, u_\alpha), \quad (T_{12}, u_\beta),$$
$$(Q_\alpha, W), \quad (\bar{\phi}_s M_{11} - \bar{\phi}_g M_g, \gamma_\alpha), \quad (M_{12}, \gamma_\beta). \quad (33)$$

Initial conditions for system (32) are:

$$t = 0: \quad \phi_2 = \phi_2^0, \quad \rho_g = \frac{p_0}{R_a\theta_0}; \quad \theta = \theta_0, \quad (34)$$

where ϕ_2^0, p_0, τ_0 are the initial concentration of a binder in the composite, initial gas pressure in pores and initial temperature. Function ϕ_3 can be determined in terms of ϕ_2 analytically:

$$\phi_3 = (\phi_2^0 - \phi_2)\rho_2/\rho_3(1-\Gamma). \quad (35)$$

4. CYLINDRICAL ABLATING SHELL

Now consider a special case of an ablating shell that can be used for a wide scope of applied problems. Let us consider an ablating shell being a rotation body (Fig. 1) with the symmetry axis z. The coordinate surface $q_3 = 0$ is assumed to be coincident with the middle surface of the shell; the meridional arc s and the azimuth angle ϕ counted off from the certain point M_0 are chosen as coordinates q_1, q_2 and $q_3 = R - r$, where r is the radius.

For the case of cylindrical shells the principal curvatures k_1 and k_2 are the constants:

$$k_1 = 0; \quad k_2 = \frac{1}{R}; \quad A_1 = 1; \quad A_2 = R. \quad (36)$$

Then equilibrium equations (19) and (20) can be simplified:

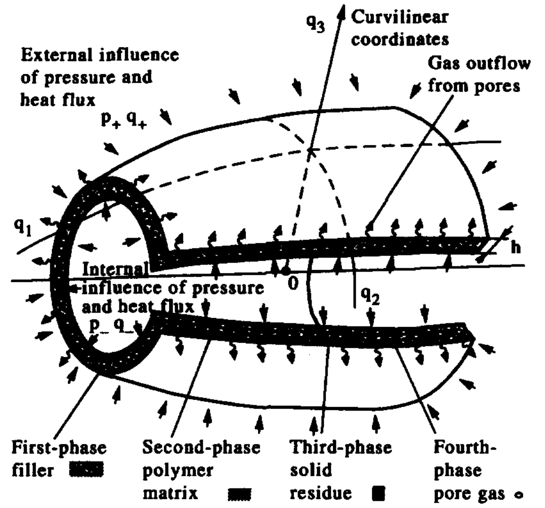

Fig. 1. Scheme of a thin-walled shell made of ablating material.

$$\frac{\partial}{\partial s}(\bar{\phi}_s r T_{11} - \bar{\phi}_g P_g) = 0;$$

$$\frac{\partial}{\partial s}(\phi_s Q_1) - \frac{\bar{\phi}_s T_{22} - \bar{\phi}_g P_g}{R} + (p_+ - p_-) = 0;$$

$$\frac{\partial}{\partial s}(\bar{\phi}_s M_{11} - \bar{\phi}_g M_g) - \bar{\phi}_s Q_1 = 0. \quad (37)$$

Kinematic relations (17) take the form:

$$e_{11} = \frac{\partial U_1}{\partial s}; \quad e_{22} = \frac{W}{R}; \quad (38)$$

$$\kappa_{11} = \frac{\partial \gamma_1}{\partial s}; \quad \kappa_{22} = 0.$$

Constitutive relations (31) for a cylindrical shell have the form:

$$\bar{\phi}_s T_{\alpha\alpha} = \bar{\phi}_s P_s + \sum_{\beta=1}^{2} C_{\alpha\beta}(h e_{\beta\beta} - \mathring{\bar{\varepsilon}}^{(0)}_{\beta\beta}) - C_{\alpha 3}\mathring{\bar{\varepsilon}}^{(0)}_{33};$$

$$\bar{\phi}_s M_{11} = -\bar{\phi}_s M_s + C_{11}\kappa_{11}\frac{h^3}{12} - \sum_{\beta=1}^{3} C_{1\beta}\mathring{\bar{\varepsilon}}^{(1)}_{\beta\beta};$$

$$\phi_s Q_1 = C_{44}\frac{h^3}{6}\left(\bar{\chi}_1\gamma_1 + \frac{h}{R}\frac{\partial W}{\partial_s}\right). \quad (39)$$

Heat and mass transfer equations (32) for a cylindrical shell are written in the form:

$$\rho_2^0 \frac{\partial \phi}{\partial t} = -J;$$

$$\partial \rho_g \partial t = \frac{1}{r}\frac{\partial}{\partial r}\left(K_{33} R_a r \frac{\partial \rho_g \theta}{\partial r}\right) + \Gamma J;$$

$$\rho c \frac{\partial \theta}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(\lambda_{33} r \frac{\partial \theta}{\partial r}\right) + c_g R_a K_{33}\frac{\partial \rho_g \theta}{\partial r}\frac{\partial \theta}{\partial r} - J\Delta e^0. \quad (40)$$

Consider the system (36) and (39) with boundary conditions of the following form:

$$s = s_+: \quad \bar{\phi}_s T_{11} - \bar{\phi}_g P_g = T_0; \quad Q_1 = 0; \quad \gamma_1 = 0;$$

$$s = s_-: \quad U_1 = 0; \quad Q_1 = 0; \quad \gamma_1 = 0. \quad (41)$$

Under these conditions the system (36)–(39) has the solution:

$$T_{11} = \frac{\bar{\phi}_g P_g}{\phi_s}; \quad T_{22} = \frac{1}{\bar{\phi}_s}((p_+ - p_-)R + \bar{\phi}_g P_g);$$

$$Q_1 = 0; \quad M_{11} = \frac{1}{\bar{\phi}_s}\left(-\bar{\phi}_s M_s - \sum_{\beta=1}^{3} C_{1\beta}\mathring{\bar{\varepsilon}}^{(1)}_{\beta\beta}\right);$$

$$\gamma_1 = 0. \quad (42)$$

Substituting formulae (42) into (39) we can find strains $\varepsilon_{\beta\beta}$, $\varepsilon_{11} = 0$. Stresses in the cylindrical shell can be determined by formulae (22), (23) and (30):

$$\sigma_{\alpha 3} = 0; \quad \alpha = 1,2; \quad \sigma_{12} = 0;$$

$$f_3 = \frac{6}{h^3}(-(\bar{\phi}_s P_s + \bar{\phi}_g P_g)\xi_1 + (p_+ - p_-)\xi_2 - C_{33}\mathring{\bar{\varepsilon}}^{(0)}_{\beta\beta});$$

$$\sigma_{\alpha\alpha} = -p + \frac{1}{\phi_s h}(\bar{\phi}_s P_s + \bar{\phi}_g P_g)$$

$$+ \frac{1}{\bar{\phi}_s}\sum_{\beta=1}^{2} C_{\alpha\beta}\left(\frac{1}{h}\mathring{\bar{\varepsilon}}^{(0)}_{\beta\beta} - \mathring{\varepsilon}_{\beta\beta}\tilde{a}_1^0\right) + (p_+ - p_-)\frac{R(\alpha - 1)}{h\phi_s};$$

$$\alpha = 1,2; \quad (43)$$

where

$$\xi_1 = 1 - \frac{1}{\Delta}(C_{31}(C_{22} - C_{12}) + C_{32}(C_{11} - C_{12}));$$

$$\xi_2 = \frac{h}{2} + \frac{R}{\Delta}(-C_{31}C_{12} + C_{32}C_{11});$$

$$\Delta = C_{11}C_{22} - C_{12}^2.$$

Heat and mass transfer equations (40) are solved numerically.

5. ABLATING SHELLS AFTER APPEARANCE OF DELAMINATIONS

Up till now structures with ablating composites have been examined only for the time interval $0 \leqslant t \leqslant t_*$, till time t_* of destruction by the thermomechanical type, for example, until the composite delamination appears due to accumulating intrapore pressure of gaseous thermodestruction products. This restriction is justified for many structures, as after arising the delamination, a hardware having for example, a plane shape (plates, unclosed shells, panels), fails and loses completely its exploitational properties. However, for some types of structures there exist exploitational conditions when the hardware still performs its functional purpose for some time interval $t_* \leqslant t \leqslant t_{**}$ after delamination appearance.

This situation takes place in external heating a shell with a closed contour made of thermodestructing composite material (Fig. 2), for example under the action of flame onto a cylindrical tank-container of glass–plastic. The action of a temperature field uniformly distributed over the surface on the cylindrical shell of the tank-container leads to forming ring delaminations having a closed contour.

Because the shell of the tank-container has a closed contour, it does not fail after arising the first delamination and continues to perform its functional purposes. In further heating, new ring delaminations appear, the formation process of which is directed from the outer surface of the shell to the inner one. Pore pressure is accumulated in each ring crack so that the pressure difference $p_d(r_i, t) - p_d(r_{i-1}, t)$ (where $i = 1, \ldots, N$, r_i is the radius of ith delamination) uniformly distributed acts upon each of the cylindrical layers stripped off. The nondelaminated part of the cylindrical shell of the tank-container, with the thickness $h_d(t)$ (Fig. 2B), proves to be the most loaded, as that undergoes the action of the maximal pressure difference $p_d(r_N, t) - p_-$, $h_d = r_N - R_1$.

The process of delamination formation continues till time t_{**} when at the certain critical thickness

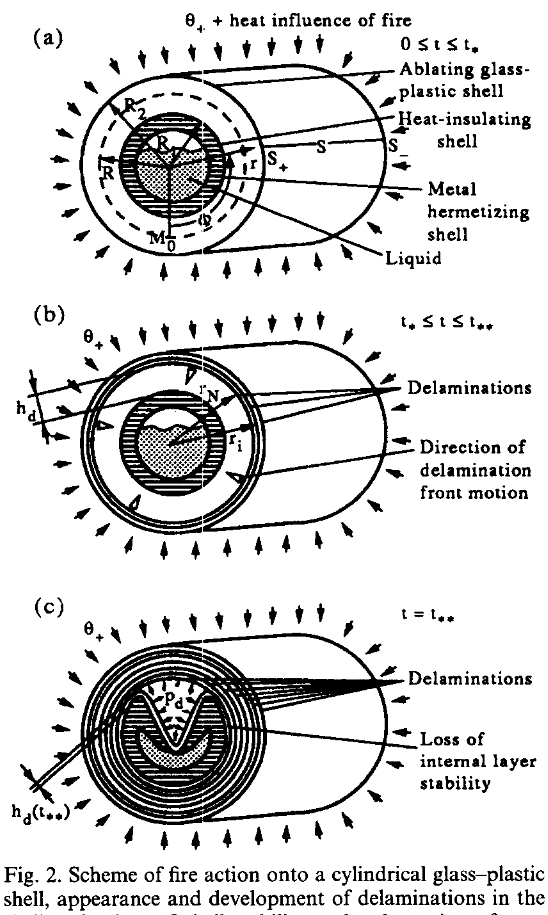

Fig. 2. Scheme of fire action onto a cylindrical glass–plastic shell, appearance and development of delaminations in the shell and a loss of shell stability under the action of pore pressure of ablation gas.

$h_d(t_{**})$ the inner part of the shell loses a stability under the action of the external pressure difference $p_d(r_N, t_{**}) - p_{e-} > 0$ (Fig. 2C). After that the structure stops to perform its functional purpose and the tank-container fails completely. However, the time interval $t_{**} - t_*$ since the first delamination appearance till a loss of stability of the tank-container shell is sufficiently long: $t_{**} - t_* \gg t_*$, therefore the method of stresses and heat–mass transfer calculations developed in Sections 2–4 for $t < t_*$ should be continued for the time interval $t_* < t < t_{**}$.

A condition of the delamination appearing in ablating composite has the form:

$$z_1(t_*(r), r) = 1, \tag{44}$$

where the damage functional z_1 can be represented by the formula

$$z_1(t, r) = \frac{|\sigma_{33}(t, r)| + \sigma_{33}(t, r)}{2\tilde{a}_1^0(t, r)\sigma_3^+}, \tag{45}$$

where σ_3^+ is the strength of laminated composite in tension along the Ox_3 direction.

For a cylindrical shell the stress σ_{33} can be determined by formula (23). Substituting the expression for σ_{33} into (44) we obtain:

$$z_1 = \frac{\phi_g(r, t_*)p(r, t_*) - p_- + (p_- - p_+)\left(\frac{r-R}{h} + \frac{1}{r}\right) + f_3(r, t_*)\left(\frac{h^2}{4} - (r-R)^2\right)}{\phi_s(r, t_*)\tilde{a}_1^0(t_*)\sigma_3^+} = 1. \tag{46}$$

From this equation the function $t = t_*(r)$ describing the advance of the delamination front in the shell can be determined. We will assume that the appearance of delamination does not change a picture of heat–mass transfer in the shell.

The critical external pressure p^*, for which the loss of stability of the nondelaminated shell section occurs, is written as follows:

$$p^*(t) = 0.92E_2\tilde{a}_1^0(t)\left(\frac{h_d(t)}{r_N}\right)^{2.5} \times \frac{r_N}{L}, \tag{47}$$

where L is the shell length and the stability condition has the form

$$p_d(r_N, t) \leqslant p^*(t) - p_-, \quad t_* \leqslant t \leqslant t_{**}. \tag{48}$$

Now, derive the expression for the pressure p_d of gaseous pyrolise products accumulated in a ring crack. Use the equation of gas mass conservation:

$$\rho_{gd}V_d = \rho_g(V_{g+} - V_{g-}), \tag{49}$$

where V_g is the gas volume in pores entering the crack for time $(t - t_*)$, ρ_{gd} is the gas density in the ring crack

$$\rho_{gd} = \frac{p_d}{R_a^\theta}; \tag{50}$$

V_d is the crack volume opened by the pressure p_d action

$$V_d = \pi L(r_N^2 - r_{N\text{def}}) = 2\pi L \frac{p_d r_N^3}{h_d E_2 \tilde{a}_1^0}; \quad (51)$$

where r_N and $r_{N\text{def}}$ are the radii of the surface of the Nth delamination in the non-opened and opened by deforming states, respectively,

$$r_{N\text{def}} = r_N(1+\varepsilon_\theta) = r_N\left(1 + \frac{p_d r_N}{h_d E_2 \tilde{a}_1^0}\right);$$

ρ_g is the gas density in pores, V_{g+} is the gas volume in pores entering the crack for time $(t-t_*)$ and V_{g-} is the gas volume outflowing from the crack for time $(t-t_*)$

$$\rho_g(V_{g+} - V_{g-}) = 2\pi RL \int_{t_*}^{t} \rho_g(v_{g+} - v_{g-})\, d\tau. \quad (52)$$

On substituting formulae (50)–(52) into (49) and writing the Darcy's relation for the crack borders:

$$\rho_g v_{g\pm} = -K_{33}\left(\frac{\partial p}{\partial r}\right)_{\pm}, \quad (53)$$

the expression for the pressure p_d is derived

$$p_d(r_N, t) = \left(K_{33} R_a E_2 \tilde{a}_1^0 \cdot \frac{h}{r_N^2} \int_{t_*}^{t} \left(\left(\frac{\partial p}{\partial r}\right)_- - \left(\frac{\partial p}{\partial r}\right)_+\right) d\tau\right)^{1/2}. \quad (54)$$

Using expressions (47), (48), (54), the condition for a loss of delamination stability at time t_{**} can be derived as follows:

$$\left(K_{33} R_a E_2 \tilde{a}_1^0 \cdot \frac{h}{R^2} \int_{t_*}^{t_{**}} \left(\left(\frac{\partial p}{\partial r}\right)_- - \left(\frac{\partial p}{\partial r}\right)_+\right) d\tau\right)^{1/2} = 0.92 E_2 \tilde{a}_1^0 \left(\frac{h_d}{r_N}\right)^{2.5} \cdot \frac{r_N}{L}, \quad (55)$$

where the time t_* is evaluated from (46).

6. COMPUTED RESULTS

To estimate the accuracy of the thin-walled ablating shells' theory developed above, computations were performed for stresses and parameters of internal heat–mass transfer of a cylindrical shell modelling a glass–plastic tank-container, the whole surface of which undergoes the action of fire. Computation was conducted in two ways: according to the faithful theory presented in Section 2 by the numerical difference scheme (this theory for cylindrical shells is described in detail in [2]) and according to the shell theory, i.e. by formulae (43) and (32).

The tank-container structure was considered in the form of a three-layered shell: the external layer was an ablating glass–plastic investigated, the middle layer—heat-insulator, the internal layer—a thin hermetic metallic shell.

All computations were conducted for a glass–plastic shell with the following geometric parameters: $R = 2$ m, $h = 2\times10^{-3}$ m, $L = 3$ m and with the following physical characteristics corresponding to the glass–plastic on the base of epoxy–phenol resin and glass fabric:

$$\lambda_{ii} = \lambda_{i(1)}\phi_1 + \lambda_{i(2)}\phi_2 + \lambda_{i(3)}\phi_3;$$

$$\rho = \rho_1\phi_1 + \rho_2\phi_2 + \rho_3\phi_3; \quad K_{ii} = K^0 \exp(-s\phi_g),$$

$$\alpha_1 = 2\times10^{-6}\,\text{K}^{-1}, \quad \beta_{sh} = 5, \quad \alpha_2 = 20\times10^{-6}\,\text{K}^{-1},$$

$$\alpha_3 = 2\times10^{-6}\,\text{K}^{-1}, \quad a_2 = a_3 = 0,1;$$

$$\rho c = \rho_1 c_1 \phi_1 + \rho_2 c_2 \phi_2 + \rho_3 c_3 \phi_3;$$

$$\rho_1 = 2.5\times10^3\,\text{kg m}^{-3}, \quad \rho_2 = 1.2\times10^3\,\text{kg m}^{-3},$$

$$\rho_3 = 2.2\times10^3\,\text{kg m}^{-3},$$

$$c_1 = 0.89\,\text{kJ kg}^{-1}\,\text{K}^{-1}, \quad c_2 = 0.6\,\text{kJ kg}^{-1}\,\text{K}^{-1},$$

$$c_3 = 1.5\,\text{kJ kg}^{-1}\,\text{K}^{-1}, \quad c_g = 3.1\,\text{kJ kg}^{-1}\,\text{K}^{-1};$$

$$\lambda_{1(3)} = 0.51\,\text{Wt m}^{-1}\,\text{K}^{-1}, \quad \lambda_{2(3)} = 0.27\,\text{Wt m}^{-1}\,\text{K}^{-1},$$

$$\lambda_{3(3)} = 0.5\,\text{Wt m}^{-1}\,\text{K}^{-1}, \quad \lambda_{g(3)} = 0.1\,\text{Wt m}^{-1}\,\text{K}^{-1};$$

$$J_0 = 3.2\times10^6\,\text{kg m}^{-3}\,K^{-1}, \quad E_a/R_a = 5.5\times10^3\,K,$$

$$\Gamma = 0.78, \quad K^0 = 1.8\times10^{-19}\,\text{s}, \quad S = 100;$$

$$\sigma_3^+ = 20\,\text{MPa}, \quad n = 5, \quad E_1 = 20\,\text{GPa},$$

$$E_2 = 20\,\text{GPa}, \quad E_3 = 2\,\text{GPa}; \quad \nu_{12} = 0.27,$$

$$\nu_{23} = 0.021, \quad \nu_{13} = 0.021, \quad G_{12} = 8\,\text{GPa},$$

$$G_{23} = 0.72\,\text{GPa}, \quad G_{13} = 0.72\,\text{GPa}.$$

External and internal pressures are considered to be atmospheric: $p_- = p_+ = 0.1$ MPa, the internal surface of the glass–plastic shell is assumed to be heat-insulated. The action of a fire flame onto the external surface of the shell is modelled by giving the temperature θ_+ in the form of the known function of time:

$$\theta = \theta_+(t).$$

Figure 8 shows the shape of this function.

The action of high temperature (max $\theta_+ = 880$°C) leads to heat propagation into the glass–plastic shell. Figure 3 shows the temperature $\theta(r, t)$ distribution along the shell thickness for different times. Figure 5 shows the corresponding distributions of coke content $\phi_3(r, t)$ in the ablating material through the shell thickness and distributions of polymer phase $\phi_2(r, t)$ for different times $t = 10, 100, 200, 300, 400, 500$ and 600 s. Figure 4 shows distribution of pore gas pressure $p(r, t)$ through the shell thickness for the same times.

The temperature $\theta(r, t)$ profile at each time moment

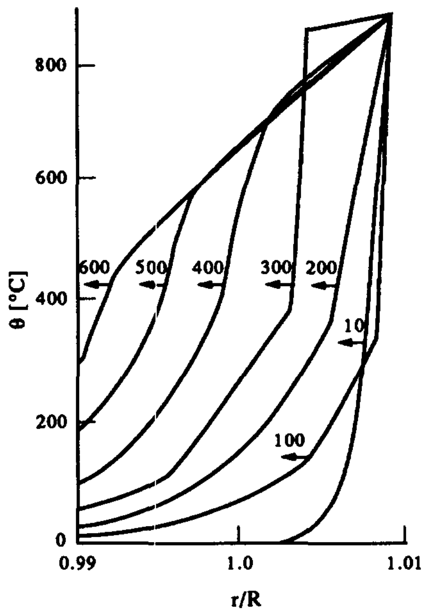

Fig. 3. Temperature θ distribution vs a thickness of a cylindrical shell made of ablating material for different times t, symbols near the curves are times t (s). Arrows show the direction of heating front propagation.

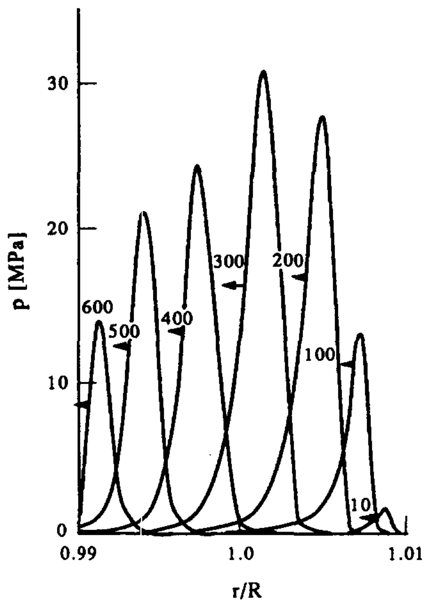

Fig. 4. Internal pore gas pressure p distribution in a cylindrical shell made of ablating glass–plastic for different times t of fire action.

has three parts, where $\theta \leqslant 150°C$, $150°C \leqslant \theta \leqslant 400°C$ and $\theta \geqslant 400°C$. The first section is defined only by heat-conductivity of non-coked glass–plastic. At the second section an intensive volumetric ablation occurs and considerable gas quantity is generated and then filtrated to the external surface of the shell. Due to the fact there is a so-called phenomenon of porous

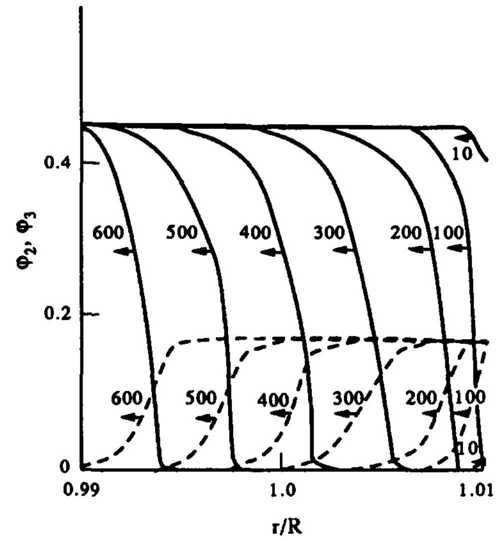

Fig. 5. Distributions of polymer phase ϕ_2 (——) and coke phase ϕ_3 (– – –) concentrations in ablating glass–plastic vs a shell thickness for different times t of fire action.

cooling, when gases filtrate through the hotter solid frame and the cool one. It is to the second section that at each time moment a peak of pore pressure corresponds (Fig. 5) and this peak is displaced in the internal surface direction following the displacement of the second section of the temperature. At the third section of temperature $\theta \geqslant 400°C$ the ablating glass–plastic is essentially coked, its porosity is so great that ablation gases are freely filtrated not creating an excess pore pressure.

Figures 6 and 7 show distributions of radial σ_{33} and tangential σ_{22} stresses in the shell for different times. Solid curves show the stresses calculated by the faithful equations of Section 2 and dashed lines correspond to the values determined by the shell theory. As seen from these figures the shell theory suggested describes all qualitative effects of the stress state in ablating material, moreover there is a good quantitative coincidence: for radial stresses distances from the exact solution do not exceed 15%, for tangential stresses these values also do not exceed 15% at the first and the second sections and only for the third section at the zone of coked material these values are more considerable and equal to 40%. However, due to the fact that the coked zone is usually eliminated from strength calculations this accuracy can be acceptable.

Peaks of radial stresses are caused by pore gas pressure generated in material ablation. Peaks of negative tangential stresses are also caused by the pore pressure. Tensile tangential in the coked zone are caused by shrinkage of the ablating material.

As seen from these figures, the most dangerous stresses are tensile radial ones. At time $t = t_* \approx 50$, the condition (46) is realized for the first time and the first shell delamination near the external surface occurs. Figure 8 exhibits the picture of appearance

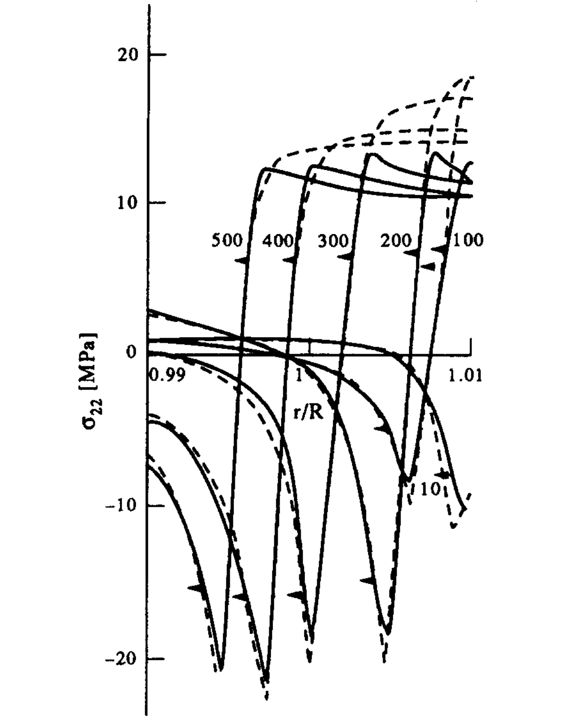

Fig. 6. Distributions of tangential stresses σ_{22} vs a thickness of a cylindrical shell made of ablating glass–plastic for different times t of fire action. Symbols near the curves are times t (s). Solid curves (——)—solution obtained by numerical integration of system (1), dashed curves (– – –)—solution (43) obtained by the shell theory.

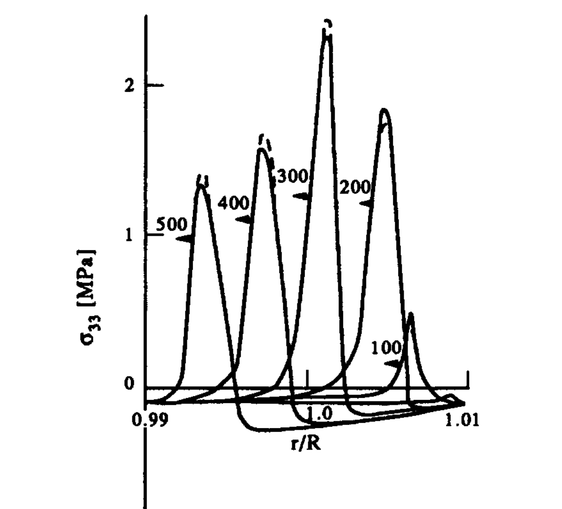

Fig. 7. Distributions of radial stresses σ_{33} vs a thickness of a cylindrical shell made of ablating glass–plastic for different times t of fire action. Symbols near the curves are times t (s). Solid curves (——)—solution obtained by numerical integration of system (1), dashed curves (– – –)—solution (43) obtained by the shell theory.

of new delaminations, this graph is characterized by thickness $h_d(t)$ of the nondelaminated material. From this figure the dependence $h_d/h(t)$ is seen to be prac-

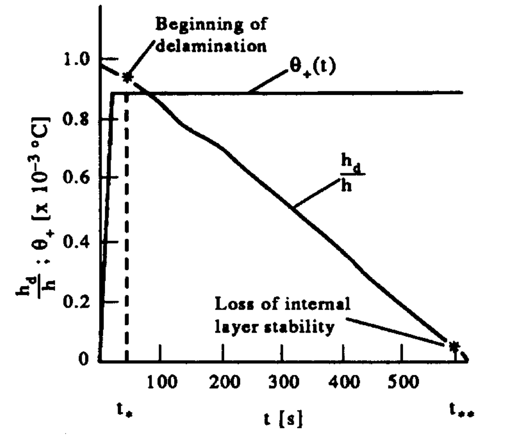

Fig. 8. Dependence of a relative thickness of nonlaminated shell part h_d/h upon time t of fire action.

tically linear. At time $t = t_{**} \approx 600$ s, when h_d/h reaches the value 0.05, the nondelaminated shell thickness become so small that pore pressure p_d of gas accumulated in delamination cavities exceeds the stability limit (55) and a stability loss of the internal layer of the glass–plastic shell occurs (Fig. 2C). In this case a complete failure of the tank-container occurs as its internal hermetizing shell is, as a rule, thin and does not resist gas pressure.

As seen from the computations conducted, the time interval $t_{**} - t_*$ is equal approximately to 550 s, that exceeds t_* by 10 times. Thus, calculation of internal heat–mass transfer and stresses is necessary up to time t_{**} as the shell still performs its designated purpose at these times.

7. CONCLUSIONS

1. The theory of heat–mass transfer processes and stresses in ablating thin-walled shell structures is developed.

2. Computational accuracy by this theory is sufficiently high (the error does not exceed ~ 15% except the coked zone where the error reaches ~ 40%). Due to the fact that this zone is not as a rule, calculated in a force scheme, such accuracy is quite acceptable.

3. The theory of delaminations' appearance and propagation in thin ablating shells and the theory of their stability loss in ablation are developed. The last theory can be successfully applied in calculation of glass–plastic shell structures' stability, for example, in tank-containers for the carriage of harsh media under conditions of the fire action.

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